The Myth of Inertia Matching

(Inertia Ratios and Why We Calculate Them)

Nick Repanich
Adjunct Research Professor
Department of Mechanical and Mechatronic Engineering and Sustainable Manufacturing
California State University, Chico

Imagine you have a 50,000 lb load and you place it on a truck, and the tires go flat. To find out what went wrong, you look at the specifications on the truck, and see that it weighs 10,000 lb. You do some quick math to find that the load weighs 5 times the truck. No wonder you say. The correct load-to-truck ratio is 1:1. So you tell your boss you need a heavier truck to haul the load, one that weighs at least 50000 lb. He gives you a strange look, but gives you a truck of the requested weight, and what do you know – the truck carries the load easily!

Did you use the correct logic to size the truck? Not really, yet this is exactly the logic used by many motion control engineers currently designing machines when it comes to sizing a motor for a particular load. How did we get here? It is an interesting tale...

Many technical reference sections found in manufacturer's catalogs discuss inertia ratios. They commonly tell you that you should try for a 1:1 match between your motor and your load. They next tell you that you should never go above a 10:1 load-to-motor inertia ratio. But the thing they never tell you is why they state this, or what their sources are. They of course do not have to, but ask the authors of those sections, and you will find no further answers. It seems as though one manufacturer read all the other manufacturer's application sections, and then wrote theirs as a compilation of the others.

There are important things to know that almost seem to be hushed up. Certainly if you ask most of the salespeople who you might buy a motion control system from, they would be able to repeat those ratios of "good design". But ask them why and they stumble. The answers are just not published. There have been some excellent articles reviewing the technical details behind the 1:1 ratio. In pure power translation, it is fairly easily shown how a 1:1 ratio gives optimum power transfer, at least in terms of efficiency. As they point out, this is rarely useful in an actual application once cost is factored into the equation, thus the 1:1 ratio merely causes one to buy bigger motors.

To get some insight into finding the right size motor for your application, imagine a simple linear motion system. A 1 kg block is to be moved back and forth 100 mm on a tabletop. Your hand is the power behind the system. In your hand is an identical 1 kg block. Initially, the two blocks are securely held together as one solid block. As you move the 1 kg block in your hand, the other block moves with you. As you accelerate your block at a rate of 1g, the other block had the exact acceleration rate as yours, as they are directly connected. As you decelerate, so does the load block. This is an ideal system with infinite stiffness and no non-linearities. You have complete control over the motion of the load without touching the load.

Now, between the load block and your block, place a very stiff 100mm spring, one similar to a car spring. Move your hand on the same manner. The load again does almost exactly the same motion as your block, because the spring you chose was so stiff. But now, again between the two blocks, place a less stiff 100mm spring, like a bed spring. Again accelerate your hand. Now as you accelerate your block, the other block does not have the same motion! Instead, as you accelerate your block, the spring begins to compress. Once the force in the spring rises (due to Hooke's Law) to a force equal to that of the force required to create the acceleration ($F=-kx=ma$), the other load begins to move similar to your hand. But you have very little control over the position of the load, which was your initial goal.

The force on the load rises to 1g as you accelerate. Now begin to decelerate your block to a stop, still in the same direction. The load does not slow at your rate. Instead, the spring begins to stretch, until it reaches a displacement from equilibrium (the same as it as when compressed). At this point, your load may be stopped, and the load may just be beginning its deceleration. Now as you stay stopped, the load begins to oscillate. It is stable (predictable) oscillation, but none the less, it is not sitting still like your hand. You could certainly not use the load's position for any automated purpose, as it is not sitting still!

Now imagine that you want to move the load back. *If you begin your acceleration anytime during the deceleration of the* *load, you will feel a force higher than 1g.* In fact, if you move at just the wrong time, you could feel up to 2g's at your hand. Why is this significant? Because when you did your sizing to find a motor to move this load, you calculated the needed force based on the *commanded* profile of your hand, not the *actual* of the load. You never predicted a force higher than 1g in your calculations. And now you have up to double that force in your system. Better hope you had plenty of torque margin. Note that just picking a larger motor is much like just picking a bigger truck. Both can be pretty expensive solutions.

So what happened? You had a 1:1 inertia ratio, supposedly the ideal ratio. You had a 50% torque margin, just what the books say. But yet your system oscillates wildly and is useless. It is because the most important factor in having control over the position of the load is *stiffness*, not its inertia ratio to the motor. You could probably imagine what would happen if your load was 100 times as large with the same wimpy spring. No control at all. But similarly, you can also imagine a load that size that is directly connected, or uses a stiff car spring. You would intuitively have quite good control over the load. Of course the system bandwidth decreased, but only because of the additional mass. $f_n=(\pi/2)(\sqrt{(k/m)}$

Inertia ratio is an indicator of possible stability problems. It is not the cause of the problem.

So why do we use the ratio all the time in sizing motors? Because *the ideal thing you would like to know in any system is the stiffness and linearity of your system*. It is simply not usually possible to determine this in a machine, especially one that does not actually exist at the time the motors must be determined and ordered. The inertia ratio is a convenient ratio to check, and does give an indication of what may happen if you design a wimpy machine.

Interestingly, other non-linearities such as backlash cause a similar reaction, (more force/torque needed than predicted), as the spring. Imagine instead of a spring, you used a short chain to move the load around. As you accelerate initially, all is fine, as it is pushing directly on the load. But when you decelerate, the load goes to the end of its tether, and gives a jerk when it stops. Say hello to a big torque spike, an ideal impulse function, the bane of any control system.

Back to the truck. What was really wrong? Do you calculate the weight of the truck and compare it to the weight of the load? Is the *ratio* the reason the tires went flat? No, but certainly the obvious solution, to either get a bigger truck, or to load it with a lesser weight did work. In fact, the ratio is only an indicator that the tires may have a flatness problem. The solution that only looks at the ratio did not factor in the type or number of tires, the type of truck, the stiffness of the frame, or the air pressure in the tires, all of which probably more directly contribute to the ability to hold the load without flat tires.

Adding a gearhead and/or ballscrew give a higher torque capability on the same motor, using more of the power of the motor by running it at a higher speed. This increases the headroom towards torque saturation, allowing for higher gains. If you go back and calculate the new inertia ratio, you find it to be a smaller ratio (by the ratio or pitch squared). But again, the ratio was not the problem or solution, only an indicator.

When you size your motors, do calculate the load-to-motor inertia ratio. Calculate the torque margin as well as the velocity margin throughout the operating range. And use your best gut feel as to the overall stiffness of the machine. Ask others with experience for their opinion as to the stiffness of the machine. And if you have a stiff machine, do not worry about having a ratio higher than 1:1. In fact there are many successful machines with relatively high bandwidths with ratios above 100:1. You can be assured they used stiff materials, including stiff couplers, which may be one of the most important “springs” in the system. After all, a machine is really just of bunch of springs in between the motor and your parts. Perhaps someday there will be a much easier way to analyze that relative stiffness. For now, we will just have to continue to calculate inertia ratios and make good guesses.

We'll Go First!

California Mechatronics Center

Chico, CA 95929-0789 | (530) 801-1983