

INTRODUCTION TO MOTOR SIZING

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SEVEN STEPS OF SIZING AND SELECTION

Step 1: Develop the torque and inertia equations that model the system mechanics.

1a: Draw/diagram the system to establish the relative location of the load mechanics.

1b: Develop the acceleration (T_a =), friction (T_f =), gravity (T_g =), and thrust (T_{th} =) torque equations. Since $T_a = (J)(\alpha)$, this will also involve developing the inertial model (J_{load} =).

Step 2: Determine the load motion profile(s) and calculate peak values.

- First find the max velocity reached (V) in a triangular move. If OK, use it. $V = \frac{2X}{t_m} = \frac{2x_a}{t_a}$
 - a simple triangular profile minimizes torque (lowers cost), but uses higher speeds
- If V is too high for any reason, then find the optimized acceleration time (t_a) in a trapezoidal move using overall move time, distance & the new constrained velocity. $t_a = t_m - \frac{X}{V}$
 - trapezoidal profiles are useful where rated motor torque drops with speed
 - for steppers, set maximum speed where the motor changes from the constant torque range into the constant power range, otherwise reduction may be necessary.
- If there are multiple move profiles, find the worst-case acceleration (a), where $a = \frac{V}{t_a}$

Step 3: Calculate the mass moment of inertias of the load mechanics.

- Anything that moves *with* the motor is part of the inertia (J).

Step 4: Determine the peak torque (T_p) at maximum speed excluding motor inertia.

- determine worst case combination of T_a , T_f , T_g , T_{th} ($T_{peak} = T_a \pm T_f \pm T_g \pm T_{th}$)
- calculate power required to move the load (to find a system in the right power range)

Step 5: Choose an approximate motor/drive system.

- find a motor/drive with more than the required speed and about double the power
- look at speed-torque curves and price list simultaneously
 - with servos, use most of the speed available or you waste the power available

Step 6: Determine the peak torque at speed including motor inertia.

- calculate RMS torque to evaluate motor heating issues (necessary for servo systems)
- give 50-100% torque margin (only use lower margins if measurable mechanics exist)
- check for 10-20% velocity margin

Step 7: Optimize the system.

- Check the load-to-motor inertia ratio, and compare to the machine's stiffness to the performance desired. A ratio higher than 10:1 is an indicator of potential instability problems with non-stiff systems. (Note: It is *not* the cause of the problem.)
- Check the need for a power dump circuit (high inertia ratios or vertical load)
- It may be necessary to adjust mechanics, add reduction, and start over. Remember, *iteration is crucial to successful design!*

MOTION PROFILE FORMULAS

for triangular profiles:

$$V = \frac{2X}{t_m} = \frac{2x_a}{t_a}$$

for trapezoidal profiles:

$$t_a = t_m - \frac{X}{V}$$

Where:

V	=	maximum velocity
X	=	total move distance
t_m	=	total move time
x_a	=	acceleration distance
t_a	=	acceleration time

Note: V is used here rather than ω , as these formulas work with either linear or rotary units.

GENERAL TORQUE & POWER FORMULAS

$$T_{peak} = T_a \pm T_f \pm T_g \pm T_{th}$$

Torque Equations:

$$T_{rms} = \sqrt{\frac{T_1^2 t_1 + T_2^2 t_2 + T_3^2 t_3 + T_4^2 t_4}{t_1 + t_2 + t_3 + t_4}}$$

Where:

T_a	=	Motor torque required to accelerate the inertial load
T_f	=	Motor torque required to overcome the frictional forces
T_g	=	Motor torque required to overcome the gravitational forces load
T_{th}	=	Motor torque required to overcome any additional thrust forces
T_1	=	Torque to accelerate the load from zero speed to max speed ($T_f + T_a$)
T_2	=	Torque to keep the motor moving once it reaches max speed ($T_a = 0$)
T_3	=	Torque required to decelerate from max speed to a stop ($T_a - T_f$)
T_4	=	Torque required while motor is sitting still at zero speed
t_1	=	time spent accelerating the load
t_2	=	time spent while motor is turning at constant speed
t_3	=	time spent decelerating the load
t_4	=	time spent while motor is at rest

Torque Unit Conversions:

$$N = \frac{kg \cdot m}{s^2} \quad \therefore \quad \frac{kg \cdot cm^2}{s^2} \left(\frac{1 m}{100 cm} \right)^2 = N \cdot m$$

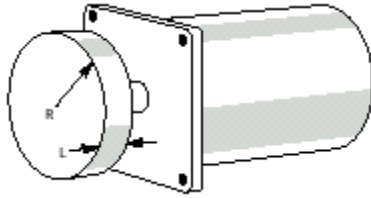
Power Unit Conversions:

$$\text{Watts} = 2\pi(N \cdot m)(rps) = \frac{2\pi(N \cdot m)(rpm)}{60} = \frac{(oz \cdot in)(rps)}{22.52} = \frac{(in \cdot lb)(rpm)}{84.45} = \frac{(ft \cdot lb)(rpm)}{7.038} = \frac{(ft \cdot lb)(rps)}{0.1173}$$

$$1 \text{ hp} = 746 \text{ W}$$

DIRECT DRIVE (ROTARY) FORMULAS

Inertia (Step 3):



Solid Cylinder

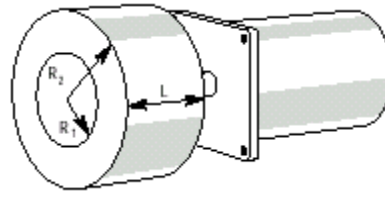
Where ρ , the density is known, use

$$J_{load} = \frac{\pi L \rho R^4}{2}$$

Where mass and radius are known use

$$J_{load} = \frac{m R^2}{2}$$

$$m = \pi L \rho R^2$$



Hollow Cylinder

Where ρ , the density, is known use

$$J_{load} = \frac{\pi L \rho}{2} (R_2^4 - R_1^4)$$

Where m , the mass, is known use

$$J_{load} = \frac{m}{2} (R_1^2 + R_2^2)$$

$$m = \pi L \rho (R_2^2 - R_1^2)$$

Torque (Step 2, 4-7)

$$T_a = (J_{load} + J_{cp} + J_m) \left(\frac{\omega_m}{t_a} \right)$$

$$T_f = (F)(R)$$

Where:

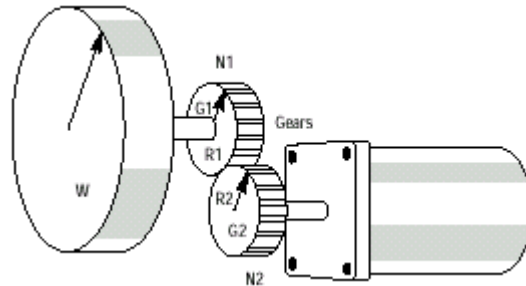
J_{cp}	=	inertia of the coupler ($\text{g} \cdot \text{cm}^2$)
J_m	=	inertia of the motor's rotor ($\text{g} \cdot \text{cm}^2$)
t_a	=	acceleration time (s)
ω_m	=	maximum motor velocity reached (rad/s)
F	=	frictional force (N)
R	=	radius at which the frictional force acts (cm)

Note 1: When using Imperial units, specifically inertia in convenient units of $\text{oz} \cdot \text{in}^2$, the acceleration torque formula must be modified to correct the units. The units of $\text{oz} \cdot \text{in}^2$ for inertia are not proper inertial units, yet they are quite convenient to calculate inertia based on real world data. To use the acceleration torque formula with inertia in $\text{oz} \cdot \text{in}^2$, divide by the gravitational constant g ($1/386 \text{ in/s}^2$), which converts $\text{oz} \cdot \text{in}^2$ to proper units of $\text{oz} \cdot \text{in} \cdot \text{s}^2$ as follows:

$$T_a = \frac{1}{386 \frac{\text{in}}{\text{s}^2}} (J_{load} + J_{cp} + J_m) \left(\frac{\omega_m}{t_a} \right)$$

Note 2: Though the correct symbol for mass moment of inertia is “ I ”, typically in mechatronics we (incorrectly) use the symbol “ J ” for inertia, as “ I ” is also used for electrical current.

DIRECT DRIVE WITH REDUCTION



Gear Drive Inertia Formulas:

$$J_{load} = \frac{m_{load} R_{load}^2}{2N^2}$$

or

$$J_{load} = \frac{\pi L_{load} \rho_{load} R_{load}^4}{2N^2}$$

$$J_{gear1} = \frac{m_{gear1} R_{gear1}^2}{2N^2}$$

$$J_{gear2} = \frac{m_{gear2} R_{gear2}^2}{2}$$

Where:

J_x	= inertia "as seen by the motor"
m	= mass
R	= radius
N	= reduction ratio ($R1/R2$)
L	= length
ρ	= density

Note: Most gearhead manufacturers provide the reflected inertia of the reducer in their user guide. This will help eliminate the need to calculate the individual gear inertias.

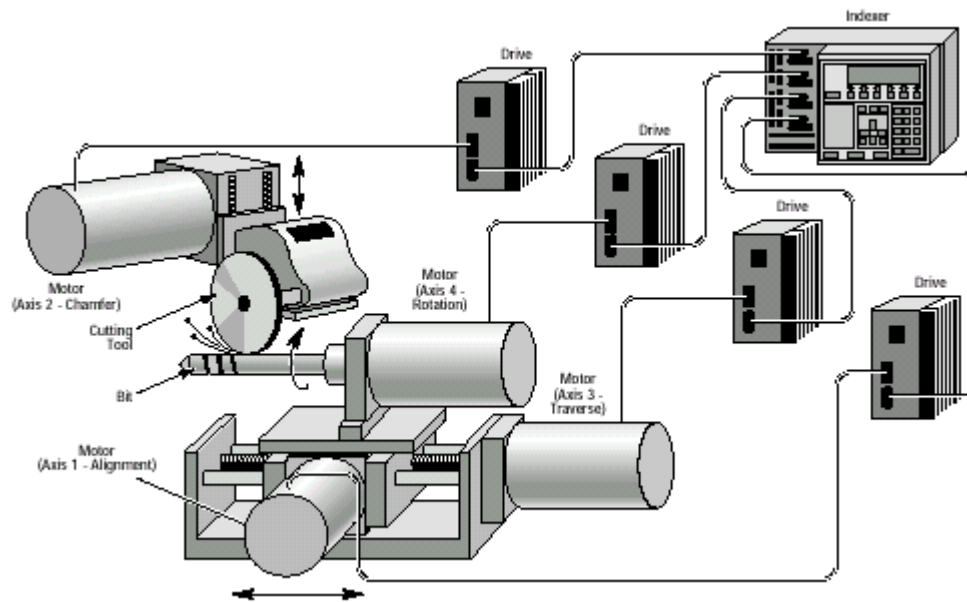
$$T_a = \left(\frac{J_{load}}{e} + J_r + J_m \right) \left(\frac{\omega_m N}{t_a} \right)$$

$$T_f = \frac{(F)(R)}{eN}$$

Where:

e	= efficiency of transmission (reduction)
N	= reduction ratio
J_r	= effective inertia of reducer, gear box, belts and pulleys, or gears
J_m	= inertia of the motor's rotor
ω_m	= maximum motor velocity reached
t_a	= acceleration time
F	= frictional force
R	= radius at which the frictional force acts

EXAMPLE #1 - Fluted-Bit Cutting Machine



Parameters for Axis 4:

- Move distance 0.2 rev
- Move time 0.15 s
- Maximum velocity allowed 20 rps
- Length of chuck 4.0 cm
- Diameter of steel chuck 3.0 cm
- Length of bit 14 cm
- Diameter of steel bit 0.8 cm
- Torque required during cutting 0.050 N·m

Step 1:

$$V = \frac{2X}{t_m} = \frac{(2)(0.2)}{0.15} = 2.67 \text{ rps}$$

(OK, since not above "knee" on most stepper curves)

Step 2:

$$T_a = (J_{bit} + J_{chuck} + J_{motor}) \left(\frac{\omega_m}{t_a} \right)$$

Step 3:

$$J_{bit} = \frac{\pi}{2} (14 \text{ cm}) (7.75 \frac{\text{g}}{\text{cm}^3}) \left(\frac{0.8 \text{ cm}}{2} \right)^4 = 4.36 \text{ g} \cdot \text{cm}^2$$

$$J_{chuck} = \frac{\pi}{2} (4 \text{ cm}) (7.75 \frac{\text{g}}{\text{cm}^3}) \left(\frac{3.0 \text{ cm}}{2} \right)^4 = 246.5 \text{ g} \cdot \text{cm}^2$$

Step 4:

$$T_a = (4.36 \text{ g} \cdot \text{cm}^2 + 246.5 \text{ g} \cdot \text{cm}^2) (2\pi) \left(\frac{2.67 \text{ rps}}{0.15 \text{ s}/2} \right) = 56,112 \text{ g} \cdot \frac{\text{cm}^2}{\text{s}^2} = 0.0056 \text{ kg} \cdot \frac{\text{m}^2}{\text{s}^2} = 0.0056 \text{ N} \cdot \text{m}$$

$$T_f = 0.050 \text{ N} \cdot \text{m}$$

$$T_{peak} = T_a + T_f = 0.0056 + 0.050 \text{ N} \cdot \text{m} = 0.0556 \text{ N} \cdot \text{m} @ 2.67 \text{ rps}$$

$$P = 2\pi(0.0556 \text{ N} \cdot \text{m})(2.67 \text{ rps}) = 0.93 \text{ W}$$

$$J_{load} = 4.36 + 246.5 = 250.86 \text{ g} \cdot \text{cm}^2$$

Step 5: Try Nema 23 half-stack step motor, $J_m = 0.070 \text{ kg} \cdot \text{cm}^2 = 70 \text{ g} \cdot \text{cm}^2$

Step 6:
$$T_a = (4.36 + 246.5 + 70) (2\pi) \left(\frac{2.67}{0.15/2} \right) = 71,770 \text{ g} \cdot \frac{\text{cm}^2}{\text{s}^2} = 0.0072 \text{ N} \cdot \text{m}$$

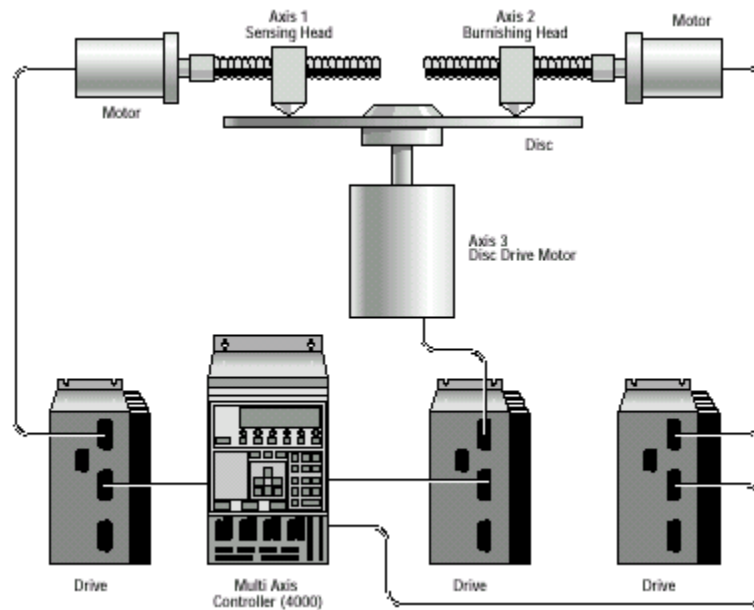
$$T_{peak} = T_a + T_f = 0.0072 + 0.050 \text{ N} \cdot \text{m} = 0.0572 \text{ N} \cdot \text{m} @ 2.67 \text{ rps}$$

Step 7:

$$\text{Inertia Ratio} = \frac{J_l}{J_m} = \frac{250.86 \text{ g} \cdot \text{cm}^2}{70 \text{ g} \cdot \text{cm}^2} = 3.6 \text{ (fine)}$$

$$\text{Torque Margin (@speed)} = \frac{T_{available} - T_{peak}}{T_{peak}} = \frac{0.26 \text{ N} \cdot \text{m} - 0.0572 \text{ N} \cdot \text{m}}{0.00572 \text{ N} \cdot \text{m}} = 355 \% \text{ (fine)}$$

PROBLEM #1 - Grinding Of Bicycle Rim Braking Surface

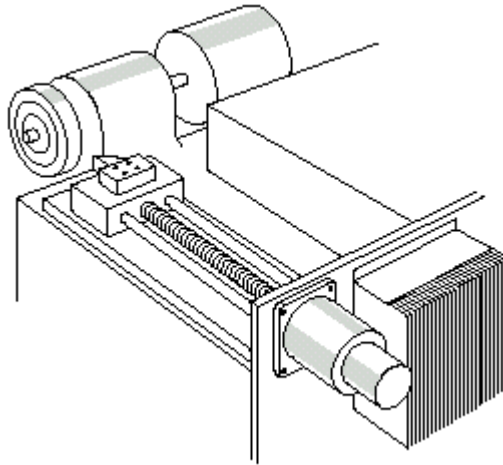


Parameters of Axis 3:

- Outside diameter of steel bicycle rim 33.0 cm
- Move distance 1/36 rev
- Move time 0.10 s
- Inertia of rim-holding fixture 1686 kg·cm²
- Width of rim 4.5 cm
- Inside diameter of rim 31.5 cm
- Friction torque during grinding 1.6 N·m
- Dwell time between moves 0.05 s
- Velocity is not limited in any way
- This is not the final grinding move, but is the worst case move, making repeated passes over the weld.

Answer: approximately 20 N·m @ 0.56 rps

LEADSCREW FORMULAS



Inertia (Step 3)

$$J_{load} = \frac{m}{(2\pi p)^2} \quad J_{screw} = \frac{\pi L \rho R^4}{2}$$

Where: m = mass (kg)
 L = length (cm)
 R = radius (cm)
 ρ = density (g/cm³)
 p = pitch of screw (revs/unit length) (inverse of lead)

The formula for J_{load} converts linear mass into the rotational inertia as reflected to the motor shaft by the lead screw.

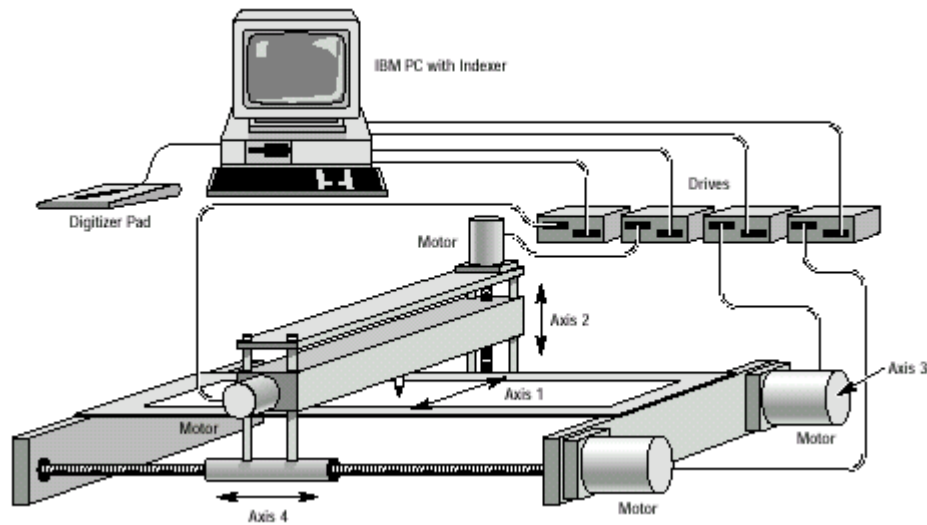
Torque (Step 2, 4-7)

$$T_a = \left(\frac{J_{load}}{e} + J_{screw} + J_{cp} + J_m \right) \left(\frac{\omega_m}{t_a} \right)$$

$$T_f = T_{br} + \frac{F}{2\pi p e} \quad T_g = \frac{mg}{2\pi p e} \quad F = \mu_d mg$$

Where: J_{screw} = inertia of lead screw or ball screw (kg·cm²)
 T_{br} = breakaway torque of nut on screw (N·m)
 F = force or thrust required
 p = pitch of screw (revs/unit length)
 e = efficiency of nut and screw
 ω_m = maximum motor speed (rev/s). (watch critical speed of screw)
 g = acceleration due to gravity
 J_{cp} = inertia of the coupling (kg·cm²)
 J_m = inertia of the motor's rotor (kg·cm²)
 t_a = acceleration time (s)
 T_g = torque required to overcome gravity (vertical)
 μ_d = coefficient of dynamic friction

EXAMPLE #3 - "Smart" Battery Inspection



Parameters for Axis 1:

- | | |
|-----------------------------|----------|
| • Length of steel leadscrew | 36 in |
| • Pitch of leadscrew | 5 |
| • Efficiency of leadscrew | 0.65 |
| • Radius of leadscrew | 0.5 in |
| • Move distance | 1.5 in |
| • Move time | 0.5 s |
| • Maximum allowed velocity | 5 in/s |
| • Weight of load | 50 lbs. |
| • Coefficient of friction | 0.01 |
| • Breakaway Torque | 25 oz·in |

Step 1:

$$V = \frac{(2) (15\text{in})}{0.5\text{sec}} = 6 \text{ in/sec, too high}$$

$$t_a = t_m - \frac{X}{V} = 0.5 - \frac{1.5}{5} = 0.2 \text{ sec}$$

$$V = 5 \frac{\text{in}}{\text{sec}} \times 5 \frac{\text{rev}}{\text{in}} = 25 \frac{\text{rev}}{\text{sec}}$$

Step 2:

$$T_a = \frac{1}{386} \left(\frac{J_{\text{load}}}{e} + J_{\text{leadscrew}} + J_{\text{motor}} + J_{\text{cp}} \right) \frac{2\pi V}{t_a}$$

Step 3:

$$J_{\text{load}} = \frac{W}{(2\pi p)^2} = \frac{50(16)}{(2\pi 5)^2} = 0.81 \text{ oz} \cdot \text{in}^2$$

$$J_{\text{leadscrew}} = \frac{\pi L \rho R^4}{2} = \frac{\pi}{2} (36)(4.48)(.5)^4 = 15.8 \text{ oz} \cdot \text{in}^2$$

$$J_{\text{cp}} = 0$$

$$T_a = \frac{1}{386} \left(\frac{.81}{.65} + 15.8 \right) \frac{2\pi(25)}{.2} = 34.69 \text{ oz} \cdot \text{in} @ 25 \text{ rps}$$

Step 4:

$$T_f = T_{br} + \frac{F}{2\pi p e} = 25 + \frac{0.01(50)(16)}{2\pi 5(.65)} = 25.39 \text{ oz} \cdot \text{in}$$

$$T_{\text{peak}} = 34.69 + 25.39 = 60.08 \text{ oz} \cdot \text{in}$$

$$\text{hp} = \frac{(60.08)25}{16,800} = 0.0894 \left(746 \frac{\text{watts}}{\text{hp}} \right) = 66.7 \text{ watts}$$

Step 5:

Choose motor with about 0.2 hp

Try S83-93, $J_m = 6.70$

Step 6:
$$T_a = \frac{1}{386} \left(\frac{.81}{.65} + 15.8 + 6.70 \right) \frac{2\pi(25)}{.2} = 48.3 \text{ oz} \cdot \text{in}$$

$$T_{\text{peak}} = 48.3 + 25.39 = 73.7 \text{ oz} \cdot \text{in} @ 25 \text{ rps}$$

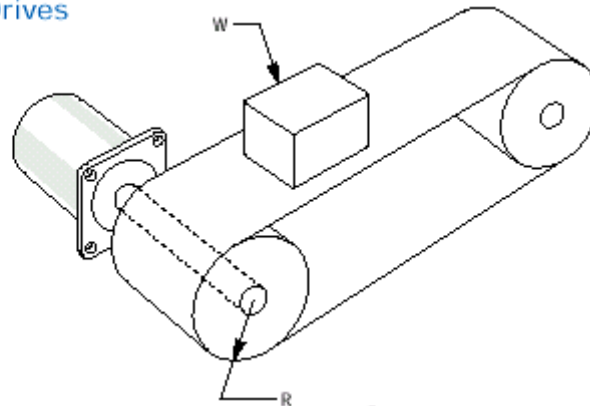
$$\frac{J}{J_m} = \frac{16.61}{6.7} = 2.48, \text{ O.K.}$$

Step 7:

Pick S83-93

TANGENTIAL DRIVES FORMULAS

Tangential Drives



Inertia Formulas:

$$J_{load} = m_l R_{dp}^2$$

$$J_{dp} = J_{fp} = \frac{m_p R_p^2}{2} = \frac{\pi L_p \rho R_p^4}{2}$$

$$J_b = m_b R_{dp}^2$$

Where:

J_{dp}	= inertia of the driving pulley
J_{fp}	= inertia of the follow pulley
J_b	= inertia of the belt
R_{dp}	= radius of the driving pulley
m_l	= mass of the load
m_p	= mass of the pulley
m_b	= mass of the belt

Belt Drive Torque Formulas:

Direct Drive:

$$T_a = \left(\frac{J_{load} + J_{belt} + J_{mechanics}}{e_b} + J_{dp} + J_m \right) \left(\frac{\omega_m}{t_a} \right)$$

$$T_f = FR_{dp}$$

With a gearhead:

$$T_a = \left(\frac{J_{load} + J_{belt} + J_{mechanics}}{e_b e_{gh} N_{gh}^2} + \frac{J_{dp}}{e_{gh} N_{gh}^2} + J_{gh} + J_m \right) \left(\frac{\omega_m N_{gh}}{t_a} \right)$$

$$T_f = \frac{FR_{dp}}{e_{gh} N_{gh}}$$

Where:

t_a	= acceleration time
e_b	= efficiency of belt over pulleys
e_{gh}	= efficiency of gearhead
N_{gh}	= reduction ratio of gearhead
$J_{mechanics}$	= inertia of other moving masses
R_{dp}	= radius of driving pulley
J_{gh}	= effective inertia of reducer
ω_m	= maximum motor speed
J_m	= inertia of the motor's rotor
F	= force or thrust or friction forces at the drive pulley

Rack & Pinion Torque Formulas:

Note: In the following formulas, the rack is stationary and the motor moves with the load. If the rack moves and the motor is stationary, then use the belt and pulley formulas, where the rack acts essentially like the belt.

Direct Drive:

$$T_a = \left(\frac{J_{load} + J_m + J_p}{e_g} \right) \left(\frac{\omega_m}{t_a} \right)$$

$$T_f = \frac{FR_p}{e_g}$$

With gearhead on motor:

$$T_a = \left(\frac{J_{load} + J_p}{e_g N^2 e_{gh}} + \frac{J_{gh} + J_m}{e_g} \right) \left(\frac{\omega_m N}{t_a} \right)$$

$$T_f = \frac{FR_p}{e_g N^2 e_{gh}}$$

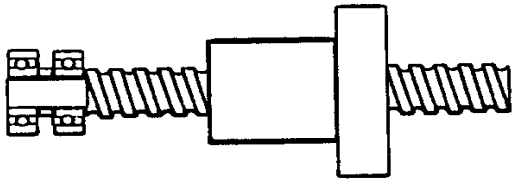
Where:

J_{load}	=	inertia of moving load
e_g	=	efficiency of transmission between rack and pinion
J_p	=	inertia of pinion gear and shafts and couplers and outboard bearings (if any)
R_p	=	effective radius of pinion gear
F	=	force, thrust or friction
J_m	=	inertia of the motor's rotor
ω_m	=	maximum motor speed
t_a	=	acceleration time
J_{gh}	=	effective inertia of reducer
e_{gh}	=	efficiency of reducer
N	=	reduction ratio

APPENDIX A

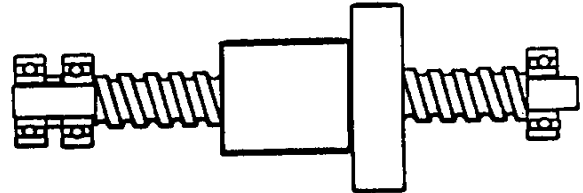
STANDARD END FIXTURING METHODS

The method of end fixturing has a direct effect on critical speed, column load bearing capacity, and system stiffness. Four common methods are shown below.



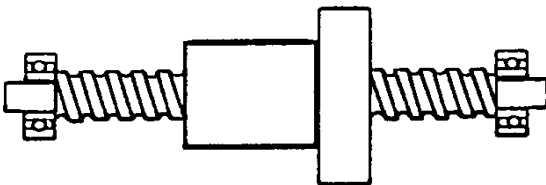
Fixed-Free

One end held in a duplex (preloaded) bearing, the other end is free



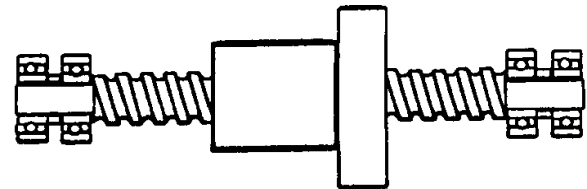
Fixed-Simple

One end held in a duplex (preloaded) bearing, one end held in a single bearing.



Simple-Simple

Both ends held in a single bearing.



Fixed-Fixed

Both ends held in duplex (preloaded) bearings.

DENSITIES OF COMMON MATERIALS

Material	oz/in ³	gm/cm ³
Aluminum Alloys	1.54	2.8
Brass, Bronze	4.80	8.6
Copper	5.15	8.9
Plastics	0.64	1.1
Steel (carbon, alloys, stainless)	4.48	7.8
Hard Wood	0.46	0.80

KINEMATIC EQUATIONS

(straight-line motion w/ constant acceleration)

Equation	Contains			
	x	v_x	a_x	t
$v_x = v_{x_0} + a_x t$		✓	✓	✓
$x = x_0 + \frac{1}{2}(v_{x_0} + v_x)t$	✓	✓		✓
$x = x_0 + v_{x_0}t + \frac{1}{2}a_x t^2$	✓		✓	✓
$v_x^2 = v_{x_0}^2 + 2a_x(x - x_0)$	✓	✓	✓	

COEFFICIENTS OF FRICTION

Materials (dry contact unless noted)	μ_s	μ_d
Steel on Steel	0.74	0.58
Steel on Steel (lubricated)	0.23	0.15
Aluminum on Steel	0.61	0.45
Copper on Steel		0.22
Brass on Steel		0.19
Teflon on Steel	0.04	0.04
Round rails w/ ball bearings	0.002	0.002
Linear guides - radius groove	0.003	0.002
Linear guides - gothic arch (depends on load, pre-load & type)	0.008 to .05	0.004 to .02

BALL SCREW FORMULAS

Critical Speed

$$N_c = \left(\frac{S \cdot d \times 10^6}{f_s \cdot L^2} \right)$$

- N_c = critical speed (rpm)
 d = root diameter of screw (cm)
 L = distance between bearings (cm)
 f_s = Safety factor (recommend 1.25)
 S = end support conditions factor
 = 4.22 fixed - free
 = 18.85 fixed - simple
 = 27.31 fixed - fixed

Note: The 'S' term carries units rev · cm /min. This equation is based on the natural frequency of the rod, and acquires its time component from that derivation.

Column Load

$$F_{cl} = \left(\frac{14.03 \times 10^6 \cdot S \cdot d^4}{L^2} \right)$$

- F_{cl} = maximum load (lb)
 S = end fixity factor
 = .250 one end fixed one free
 = 1.0 both ends supported (simple)
 = 2.0 one fixed one simple
 = 4.0 both fixed
 d = root diameter of screw
 L = distance between ball nut and load-carrying bearing

Note: The 'S' term carries hidden units that make the formula simpler and easier to use.

Backdriving Torque

(mainly used to determine holding brake torque)

$$T_b = \frac{F \times l \times e}{2\pi}$$

- T_b = torque required to backdrive
 F = axial load
 l = lead of screw
 e = efficiency of screw

Life Expectancy

For Other then Rated (Dynamic) Load

$$\text{Life} = \frac{10^6 \text{ inches}}{(\text{operating load/dynamic load})^3}$$

$$= \frac{10^6 \text{ inches}}{(F_m / \text{dynamic load})^3}$$

For Equivalent Load

$$F_m = \sqrt[3]{Y_1(F_1)^3 + \dots Y_n(F_n)^3}$$

- F_m = equivalent load
 F_n = a particular increment of load
 Y_n = the portion of a cycle (sub cycles) of a particular increment of load expressed as a decimal, i.e. the sum of the sub cycles must equal one.
 Example if L_1 is applied for 20% of the cycle, L_2 is applied for 30% of the cycle and L_3 is applied for 50% of the cycle, then the associated Y values are $Y_1 = .2$, $Y_2 = .3$, $Y_3 = .5$.

